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SPECTRAL ANALYSIS OF GAS BEARING
SYSTEMS FOR STABILITY STUDIES

by

C. H. T. Pan

December 15, 1964

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ABSTRACT

Stability map for a gas-bearing supported rotor can be constructed from the
periodic dynamic perturbation solution of the gas bearing equation about the
equilibrium condition. A complete frequency range of the perturbation solution
is required. Effective system stiffness and damping can be defined in terms of
the perturbation solution of the gas bearing. The condition of neutral stability
corresponds to the vanishing of the effective damping, yielding the characteristic
frequency, and resonance at the characteristic frequency with the equivalent
stiffness, yielding the critical mass. An excess of rotor mass causes instability
if the effective damping increases with frequency, at the characteristic frequency,
and conversely. For unloaded, journal bearings possessing rotation ¼ symmetry,
the stability map can be constructed from their steady-whirl solutions. An example
illustrating the latter case is given in terms of a herringbone-grooved journal
bearing. This method of stability analysis is applicable to both thrust and
journal bearings for both whirl and pneumatic-hammer instabilities.

INTRODUCTION

Certain types of fluid film bearings are capable of causing some form of dynamic instability of the bearing-rotor system. The best known example is the whirl instability of self-acting journal bearing, which first became understood for oil-lubricated journal bearings in Poritsky's "Theory of Oil Whip" (Ref. 1). The same phenomenon also exists with self-acting gas-lubricated journal bearings and has been referred to as "half-frequency whirl" or "fractional-frequency whirl". Although experimental data on the whirl instability of rotors supported in gas journal bearing (Refs. 2, 3, 4, and 5) are quite similar to those of rotors supported in oil journal bearings, attempts to extend the "oil-whip theory" to gas-bearing systems encountered considerable difficulty. This is primarily because the gas-lubrication equation explicitly contains the time derivative of gas density (Ref. 6). While an analogy of this equation with the heat diffusion problem is quite apparent (Refs. 7 and 8), the significance of this term in the dynamics of rotor-bearing systems has been an object of controversy for some time.

From the standpoint of obtaining the complete mathematical solution of the gas-lubrication equation, it is beyond doubt that an initial condition should be specified. On the otherhand, it is intuitively plausible that the stability of the rotor-bearing system should not have a history dependence. Earlier works on the subject reflected the latter view and neglected the time dependent term from the gas-lubricated equation altogether (Refs. 9 and 10); clearly, the accuracy of these results are not a priori known. Subsequently, it has been shown that time dependence of the gas-film pressure can be separated into an "initial transient", which is directly related to the specific initial condition and would always attenuate, and an "implicit time-dependent" term, which interacts with the motion of the rotor and controls the dynamic stability of the system (Ref. 8). Also, there is a formal method to uncouple the time-dependent gas-lubrication equation from the dynamical equation of the rotor (Ref. 8). This method of uncoupling is equivalent to a Taylor series expansion in the time coordinate and, therefore, is subject to truncation error which is not readily controllable and can significantly affect the stability analysis (Ref. 11). More recently

the stability of a rigid rotor in plain, cylindrical, self-acting, gas-lubricated, journal bearings have been studied quite thoroughly with the time-dependent effects properly considered (Refs. 12, 13 and 14). Here, peculiarities in the stability analysis of gas-bearing systems are again manifested. Where the method of Galerkin is utilized, the characteristic equation of the system (Refs. 12 and 14) is a polynominal, and Routh-Hurnitz criteria can be used to determine the condition of stability. Since the accuracy of the method of Galerkin is improved by an increased number of terms used, the corresponding characteristic equation also becomes of a higher order. Where the finite-difference method is employed (Ref. 13), the state of periodic motion, which borders the domain of unstable operation, can be found. In this analysis, one does not know, except with the additional effort of orbit calculation whether one or the other side of the neutral state is stable.

Now, stability problems are not limited to the self-acting plain journal bearings. The externally-pressurized gas-lubricated thrust bearing can exhibit a form of self-excited oscillation commonly called "pneumatic hammer" (Ref. 15), which is also possible for an externally-pressurized journal bearing. External-pressurization can reduce but will not eliminate the hazard of whirl-instability of high-speed journal bearings (Refs. 16, 17 and 18). A variation of "pneumatic hammer" has been discovered for the spiral-grooved, self-acting, gas-lubricated thrust bearing (Ref. 19). Many "whirl-free" journal bearing configurations have been proposed (Refs. 20, 21, 22, and 23).

The importance of stability analysis in the application of gas bearings has already attracted attention. However, a general method for performing the stability analysis, which is convenient to use, suggestive in physical meaning, free from arbitrary assumptions, and independent of the particulars of the bearing configuration, is still missing from public literature. The present work is intended to fill this gap, at least in part.

It will be shown, that the periodic dynamic-perturbation forces of the gas bearing can be used to construct the effective damping and stiffness coefficients of the dynamical system, that the state of neutral stability occurs at a characteristic frequency which is determined by requiring the effective damping coefficient to vanish and when the rotor mass has a critical value which is in resonance with

the effecrive stiffness at the characteristic frequency, and that an increment of the rotor mass beyond the critical value will cause instability if and only if the effective damping coefficient increases with frequency (at the characteristic frequency). Thus, the key to this method of stability analysis is the knowledge of the bearing reactions to perturbed periodic motions for the entire frequency spectrum.

Derivations will be carried out for a single mass supported in a gas bearing having two degrees of freedom. Simplifications for the special cases of a single degree of freedom bearing (e.g., thrust bearing) and an isotropic bearing (e.g., unloaded journal bearing) will also be given. To illustrate the application of this method, the stability map of an unloaded herring-bone grooved-journal bearing will be constructed.

A GENERAL THEOREM FOR THE STABILITY OF A RIGID ROTOR IN FIXED JOURNAL BEARING

The gas lubrication equation for a given journal bearing has a solution dependent on the motion of the journal. Consider Fig. 1. Let the instantaneous displacement of the journal from its equilibrium position be (e_x, e_y). Assume

$$e_x{}^2 + e_y{}^2 << c^2 \tag{1}$$

$$\frac{1}{c}(e_x, e_y) = \mathcal{Re}\left\{(\epsilon_x, \epsilon_y) \exp(s\tau)\right\} \tag{2}$$

where,

$$\tau = \omega t$$

$$s = \lambda + i\nu \tag{3}$$

Then the lubrication equation can be linearized and yield, as final results, the perturbed dynamic reaction of the bearing to the journal motion. It is customary to make the bearing reaction non-dimensional with the normalizing factor $p_a LD$. Linearization leads to the useful simplification that the perturbed bearing reaction is directly proportional to the motion. Thus, we can write in Argand notation the matrix equation *

$$f = Z(s)\epsilon \tag{4}$$

where, f is the column vector $\begin{bmatrix} f_x \\ f_y \end{bmatrix}$

---

*The homogeneous form of Eq. (4) implies that the initial transient of the fluid film pressure has been neglected. For justification of this assumption, see Ref. 8.

which is related to the actual components of the perturbed bearing reaction according
to

$$\frac{1}{p_a LD} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = Re \left\{ e^{s\tau} \begin{bmatrix} f_x \\ f_y \end{bmatrix} \right\} \tag{5}$$

$Z(s)$ is a square matrix $\begin{bmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{bmatrix}$

which may be regarded as the mechanical impedance of the journal bearing. ϵ is
the column vector $\begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix}$.

In the absence of other constraints, the equation of motion of the rotor is

$$\left[m s^2 I + Z(s) \right] \epsilon = 0 \tag{6}$$

where I is the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Because the system given by Eq. (6) is homogeneous, the determinant must vanish,

$$\left| ms^2 I + Z(s) \right| = 0 \tag{7}$$

where
$$m = \frac{MC\omega^2}{p_a LD}$$

The roots of $s = \lambda + iv$ for Eq. (7) represent the natural exponential constants
of the system. The system is stable or unstable, depending on whether v is less
or greater than zero.

Expanding Eq. (7):

$$(ms^2 + Z_{xx})(ms^2 + Z_{yy}) - Z_{xy}Z_{yx} = 0 \tag{8}$$

For convenience, introduce the definition

$$z = u + iv = -ms^2 \tag{9}$$

$$Z(s) = U + iV \tag{10}$$

Then Eq. (8) becomes

$$W\left\{z(s), s\right\} = W_1 + iW_2 \tag{11}$$

$$= (Z_{xx} - z)(Z_{yy} - z) - Z_{xy}Z_{yx} = 0$$

Separating real and imaginary parts *

$$W_1 = (U_{xx}-u)(U_{yy}-u) - (V_{xx}-v)(V_{yy}-v) - U_{xy}U_{yx} + V_{xy}V_{yx} = 0$$

$$W_2 = (U_{xx}-u)(V_{yy}-v) + (V_{xx}-v)(U_{yy}-u) - U_{xy}V_{yx} - U_{yx}V_{xy} = 0 \tag{12}$$

At the state of neutral stability

$$s = i\nu_o \tag{13}$$

$$u = m_o\nu_o^2$$

$$v = 0$$

By analogy with a single-degree-of-freedom problem, u and v can be defined as the effective spring and damping of the system.

---

*Alternately, Equation (11) can be solved for z:

$$z = \frac{1}{2}\left\{Z_{xx}+Z_{yy} \pm \sqrt{(Z_{xx}-Z_{yy})^2 + 4Z_{xy}Z_{yx}}\right\}$$

which, however, offers no advantage over Eq. (12) from the standpoint of ease in computation.

Substituting into Eq. (12):

$$W_1(0,\nu_o) = (U_{xx} - m_o\nu_o^2)(U_{yy} - m_o\nu_o^2) - V_{xx}V_{yy} - U_{xy}U_{yx} + V_{xy}V_{yx} = 0 \quad (14)$$

$$W_2(0,\nu_o) = (U_{xx} - m_o\nu_o^2)V_{yy} + (U_{yy} - m_o\nu_o^2)V_{xx} - U_{xy}V_{yx} - U_{yx}V_{xy} = 0 \quad (15)$$

Solving Eq. (15) for m_o:

$$m_o = \frac{U_{xx}V_{yy} + U_{yy}V_{xx} - U_{xy}V_{yx} - U_{yx}V_{xy}}{\nu_o^2 (V_{xx} + V_{yy})} \quad (16)$$

Substituting into Eq. (14):

$$W_1(0,\nu_o) = \frac{\left[(U_{xx}-U_{yy})V_{xx} + U_{xy}V_{yx} + U_{yx}V_{xy}\right]\left[-(U_{xx}-U_{yy})V_{yy} + U_{xy}V_{yx} + U_{yx}V_{xy}\right]}{(V_{xx} + V_{yy})^2}$$

$$- V_{xx}V_{yy} - U_{xy}U_{yx} + V_{xy}V_{yx}$$

$$= 0 \quad (17)$$

Since all elements of U and V matrices can be obtained as functions of ν by solving the pertinent lubrication equation, Eq. (17) can yield roots of ν_o; which are the characteristic frequencies. With the matrix elements evaluated at ν_o, Eq. (16) gives the critical mass, m_o, for each state of neutral stability.

For a slight variation from the state of neutral stability, writing

$$s = i\nu_o + \delta s$$

$$m = m_o + \delta m \quad (18)$$

δm and δs are related by the condition that δW vanishes. That is

$$\delta s = \delta m \left/ \frac{dm}{ds} \right|_{\delta W = 0} \tag{19}$$

Differentiating Eq. (9),

$$d z = - s^2 dm - 2ms \, ds \tag{20}$$

Treating z as an analytic function given by Eq. (11), then we can write

$$\frac{dm}{ds} \bigg|_{\delta W = 0} = - \frac{1}{s^2} \left(\frac{dz}{ds} \bigg|_{\delta W = 0} + 2ms \right)$$

Because Z is directly known in terms of ν, we can write

$$\frac{dz}{ds} \bigg|_{\delta W = 0} = \frac{1}{i} \frac{\partial z}{\partial \nu} \bigg|_{\delta W = 0}$$

Then at $s = i\nu_o$

$$\frac{dm}{ds} \bigg|_{\delta W = 0, \; s = i\nu_o} = \frac{1}{i\nu_o^2} \left[\left(\frac{\partial z}{\partial \nu} \right) \bigg|_{\delta W = 0, \; \nu_o} \cdot 2m_o \nu_o \right]$$

$$= \frac{1}{i\nu_o^2} \frac{\partial}{\partial \nu} \left(z - m_o \nu^2 \right) \bigg|_{\delta W = 0, \; \nu_o} \tag{21}$$

Substituting into Eq. (19), for slight variation from the state of neutral stability

$$\delta s = \frac{i\nu_o^2 \, \delta m}{\dfrac{\partial}{\partial \nu} \left(z - m_o \nu^2 \right) \bigg|_{\delta W = 0, \; \nu_o}}$$

Separating real and imaginary parts,

$$\delta\lambda + i\delta\nu$$

$$= \nu_o^2 \left[\frac{\frac{\partial v}{\partial \nu} + i \frac{\partial}{\partial \nu}(u - m_o \nu^2)}{\left\{ \frac{\partial}{\partial \nu}(u - m_o \nu^2) \right\}^2 + (\frac{\partial v}{\partial \nu})^2} \right]_{\delta W = 0, \ \nu_o} \delta m \tag{22}$$

The last expression is an explicit stability criterion which may be stated as follows:

With an infinitessimal mass increment beyond that determined from the state of neutral stability, the system becomes unstable when and only when $\partial v/\partial \nu$ evaluated at ν_o is positive. Conversely, if $\partial v/\partial \nu$ is negative at ν_o, the system would become unstable with a mass decrement.

To find $\partial u/\partial \nu$ and $\partial v/\partial \nu$, differentiate Eq. (11) with respect to ν:

$$\frac{\partial}{\partial \nu}(Z_{xx}Z_{yy} - Z_{xy}Z_{yx}) - z \frac{\partial}{\partial \nu}(Z_{xx} + Z_{yy}) + \left[2z - (Z_{xx} + Z_{yy}) \right] \frac{\partial z}{\partial \nu} = 0 \tag{23}$$

$$\frac{\partial z}{\partial \nu} = \frac{z \frac{\partial}{\partial \nu}(Z_{xx} + Z_{yy}) - \frac{\partial}{\partial \nu}(Z_{xx}Z_{yy} - Z_{xy}Z_{yx})}{2z - (Z_{xx} + Z_{yy})}$$

Separating real and imaginary parts

$$\frac{\partial u}{\partial \nu} + i \frac{\partial v}{\partial \nu} = \frac{(q\xi + r\eta) + i(q\eta - r\xi)}{(q)^2 + (r)^2} \tag{24}$$

where

$$q = \mathcal{Re}\left\{2z - (Z_{xx} + Z_{yy})\right\}$$

$$= 2u - (U_{xx} + U_{yy})$$

$$r = \mathcal{Im}\left\{2z - (Z_{xx} + Z_{yy})\right\}$$

$$= 2v - (V_{xx} + V_{yy})$$

$$\xi = \mathcal{Re}\left\{z\frac{\partial}{\partial\nu}(Z_{xx}+Z_{yy}) - \frac{\partial}{\partial\nu}(Z_{xx}Z_{yy}-Z_{xy}Z_{yx})\right\} \tag{25}$$

$$= u\frac{\partial}{\partial\nu}(U_{xx}+U_{yy}) - v\frac{\partial}{\partial\nu}(V_{xx}+V_{yy}) - \frac{\partial}{\partial\nu}(U_{xx}U_{yy}-V_{xx}V_{yy}-U_{xy}U_{yx} + V_{xy}V_{yx})$$

$$\eta = \mathcal{Im}\left\{z\frac{\partial}{\partial\nu}(Z_{xx}+Z_{yy}) - \frac{\partial}{\partial\nu}(Z_{xx}Z_{yy}-Z_{xy}Z_{yx})\right\}$$

$$= u\frac{\partial}{\partial\nu}(V_{xx}+V_{yy}) + v\frac{\partial}{\partial\nu}(U_{xx}+U_{yy}) - \frac{\partial}{\partial\nu}(U_{xx}V_{yy}+V_{xx}U_{yy}-U_{xy}V_{yx}-V_{xy}U_{yx})$$

Single-Degree-of-Freedom Systems

While above derivations were performed for two-degree-of-freedom systems, the results are valid for single-degree-of-freedom systems. In single-degree-of-freedom systems; e.g., parallel thrust plates, and a non-spinning externally pressurized journal bearing,

$$Z_{xy} = Z_{yx} = 0 \tag{26}$$

Equation (6) becomes uncoupled. Writing Z for either Z_{xx} or Z_{yy}, Eq. (7) is simply

$$ms^2 + Z = 0 \tag{27}$$

The state of neutral stability is then given by

$$V(\nu_o) = 0 \qquad (28)$$

$$m_o \nu_o^2 = U(\nu_o) \qquad (29)$$

A slight variation from the state of neutral stability would cause

$$\delta s = \delta\lambda + i\delta\nu$$

$$= \frac{i\nu_o^2 \, \delta m}{\left. \frac{\partial}{\partial\nu}(Z - m_o\nu^2) \right|_{\nu = \nu_o}}$$

$$= \nu_o^2 \left[\frac{\frac{\partial V}{\partial \nu} + i \frac{\partial}{\partial\nu}(U - m_o\nu^2)}{\left\{ \frac{\partial}{\partial\nu}(U - m_o\nu^2) \right\}^2 + \left(\frac{\partial V}{\partial\nu} \right)^2} \right]_{\nu_o} \delta m . \qquad (30)$$

U and V clearly have similar connotations as stiffness and damping coefficients although they may be ν dependent functions. Above results have previously been given in connection with the analysis of spiral-grooved thrust bearings in Ref. (19).

Isotropic Bearings

For those bearings, where rotational symmetry prevails,

$$Z_{xx} = Z_{yy} = Z_{/\!/} \qquad (31)$$

$$Z_{xy} = -Z_{yx} = Z_\perp \qquad (32)$$

All unloaded cylindrical journal bearings are in this category.

In Ref. (24), it was shown that reactions of an isotropic bearing caused by a small periodic motion of the journal are linearly related to those due to a small-orbit steady whirl of the journal. Referring to Fig. 2, the bearing reactions of the steady whirl problem can be written in the complex notation as follows:

$$(F_R + iF_T) = \epsilon \, F \, (\Lambda*) \tag{33}$$

$$= \epsilon \, (f + ig)$$

where $F(\Lambda*) = f + ig$ is a complex, analytical function of

$$\Lambda* = \Lambda(1-2\alpha') \tag{34}$$

$\alpha' = \pm \nu$ respectively for the forward and backward whirl cases. The impedance components are related to the steady whirl forces in the following manner (see Ref. (24) for details):

$$z_{\|} = \frac{1}{2}\left[F_c(\alpha' = \nu) + F(\alpha' = -\nu)\right] \tag{35}$$

$$z_{\perp} = \frac{1}{2}\left[F_c(\alpha' = \nu) - F(\alpha' = -\nu)\right] \tag{36}$$

where F_c is the conjugate of F.

The characteristic equation is reduced from Eq. (11) to

$$(z - z_{\|})^2 + z_{\perp}^2 = 0 \tag{37}$$

Expanding and making use of Eqs. (35) and (36):

$$z^2 - \left\langle F_c(\alpha' = \nu) + F(\alpha' = -\nu)\right\rangle \, z + F_c(\alpha' = \nu) \, F(\alpha' = -\nu) = 0 \tag{38}$$

At the state of neutral stability

$$v = v_o$$

$$z = u_o = m_o v_o^2$$

Then Eq. (38) becomes:

$$0 = u_o^2 - u_o \left[f(v_o) + f(-v_o) \right] + f(v_o)f(-v_o) + g(v_o)g(-v_o)$$

$$+ i \left\{ u_o \left[g(v_o) - g(-v_o) \right] + f(v_o)g(-v_o) - f(-v_o)g(v_o) \right\} \tag{39}$$

From the imaginary part of Eq. (39),

$$u_o = m_o v_o^2$$

$$= - \frac{f(v_o)g(-v_o) - f(-v_o)g(v_o)}{g(v_o) - g(-v_o)} \tag{40}$$

Substituting into the real part of Eq. (39):

$$0 = \left[f(v_o)g(-v_o) - f(-v_o)g(v_o) \right] \left[f(v_o)g(-v_o) - f(v_o)g(v_o) \right.$$

$$\left. + \left\langle g(v_o) - g(-v_o) \right\rangle \left\langle f(v_o) + f(-v_o) \right\rangle \right]$$

$$+ \left[g(v_o) - g(-v_o) \right]^2 \left[f(v_o) f(-v_o) + g(v_o)g(-v_o) \right]$$

$$= \left[f(v_o)g(-v_o) - f(-v_o)g(v_o) \right] \left[f(v_o)g(v_o) - f(-v_o)g(-v_o) \right]$$

$$+ \left[g(v_o) - g(-v_o) \right]^2 \left[f(v_o)f(-v_o) + g(v_o)g(-v_o) \right]$$

$$= g(v_o)g(-v_o) \left[\left\langle f(v_o) - f(-v_o) \right\rangle^2 + \left\langle g(v_o) - g(-v_o) \right\rangle^2 \right] \tag{41}$$

The solution of Eq. (41) is determined by the vanishing of either of three factors. Vanishing of the first two factors require that

$$g(\pm v_o) = 0 \tag{42}$$

Substituting into Eq. (40)

$$m_o v_o^2 = f(\pm v_o) \tag{43}$$

Vanishing of the third factor leads to

$$f(v_o) - f(-v_o) = g(v_o) - g(-v_o) = 0 \tag{44}$$

This condition renders Eq. (40) indeterminant. Therefore, m_o has to be found by other means. Substitute Eq. (44) into the real part of Eq. (39), it is found

$$\left[u_o - f(\pm v_o)\right]^2 + \left[g(\pm v_o)\right]^2 = 0$$

Therefore

$$g(\pm v_o) = 0$$

$$u_o = m_o v_o^2 = f(\pm v_o)$$

These turned out to be merely repetition of Eq. (42) and (43).

To determine whether or not deviation from the neutral state of stability in some manner would cause instability, we must know the sign of $\partial v/\partial v$ at v_o.

Differentiate Eq. (38) with respect to ν:

$$\left[2z - F_c(\nu) - F(-\nu)\right] \frac{\partial z}{\partial \nu} - z \frac{\partial}{\partial \alpha'} \left[F_c(\alpha' = \nu) - F(\alpha' = -\nu)\right]$$

$$+ F(-\nu) \left.\frac{\partial F_c}{\partial \alpha'}\right|_{\alpha' = \nu} - F_c(\nu) \left.\frac{\partial F}{\partial \alpha'}\right|_{\alpha' = -\nu} = 0 \qquad (45)$$

Substituting Eqs. (42) and (43) into Eq. (45),

$$\left[f(\pm \nu_o) - f(\mp \nu_o) \mp ig(\mp \nu_o)\right] \frac{\partial z}{\partial \nu}$$

$$- f(\pm \nu_o) \frac{\partial}{\partial \alpha'} \left[F_c(\alpha' = \nu) - F(\alpha' = -\nu)\right]$$

$$+ f(-\nu_o) \left.\frac{\partial F_c}{\partial \alpha'}\right|_{\alpha' = \nu_o} - f(\nu_o) \left.\frac{\partial F}{\partial \alpha'}\right|_{\alpha' = -\nu_o}$$

$$+ ig(\mp \nu_o) \frac{\partial}{\partial \alpha'} \left[f(\alpha' = \pm \nu) \mp ig(\alpha' = \pm \nu)\right] = 0$$

$$\left[f(\pm \nu_o) - f(\mp \nu_o) \mp ig(\mp \nu_o)\right]\left[\frac{\partial z}{\partial \nu} \mp \frac{\partial}{\partial \alpha'} \left\{f(\alpha' = \pm \nu) \mp ig(\alpha' = \pm \nu)\right\}\right] = 0$$

$$\therefore \left.\frac{\partial z}{\partial \nu}\right|_{\nu_o} = \pm \left.\frac{\partial f(\alpha' = \pm \nu)}{\partial \alpha'}\right|_{\nu = \nu_o} - i \left.\frac{\partial g(\alpha' = \pm \nu)}{\partial \alpha'}\right|_{\nu = \nu_o}$$

Separating real and imaginary parts

$$\left.\frac{\partial u}{\partial \nu}\right|_{\nu_o} = \pm \left.\frac{\partial f}{\partial \alpha'}\right|_{\alpha' = \pm \nu_o} \qquad (46$$

$$\left.\frac{\partial v}{\partial \nu}\right|_{\nu_o} = - \left.\frac{\partial g}{\partial \alpha'}\right|_{\alpha' = \pm \nu_o} \qquad (47)$$

Using these results in Eq. (22), one sees that the stability of an isotropic bearing can be evaluated in terms of its steady-whirl properties. Summarizing Eqs. (42), (43), and (47), a theorem on the stability of an isotropic bearing can be stated as follows.

Let the whirl ratio be α'. Let the radial and tangential components of the bearing reaction due to the whirl motion be F_R and F_T respectively. Let ϵ, which is assumed to be small, be the ratio between the whirl radius and the radial bearing clearance. Then the state of neutral stability is given by

$$\frac{1}{\epsilon} F_T (\alpha'_o) = 0, \tag{48}$$

which concerns the bearing only and determines α'_o; and also by

$$m_o = \frac{F_R(\alpha'_o)}{\epsilon(\alpha'_o)^2}, \tag{49}$$

which is called the critical mass.

Let the rotor mass deviate from m_o by a slight amount δm. The system is stable if and only if

$$\left. \frac{\partial F_T}{\partial \alpha'} \right|_{\alpha'_o} \delta m > 0.$$

Thus, comparing with the theorem stated below Eq. (22), $-F_T$ may be regarded as the effective damping while α' now assumes the role of ν. Note, however, α' takes on both positive and negative values while ν is always positive by definition.

PROCEDURE OF STABILITY ANALYSIS

The essence of the theorem, and its simplified versions, derived above, will be reiterated in the form of a procedure of stability analysis. The dynamical model considered here will be the translational mode of a rigid rotor supported in a fixed journal bearing.

The gas lubrication equation can be first solved for the equilibrium position of the journal designated by e_o as shown in Fig. 1 along with other parameters describing the operating conditions, e.g., Λ and L/D. Then, with the journal position perturbed by the periodic motion

$$\frac{1}{C}(e_x, \ e_y) = R_e \left\{ (\epsilon_x, \ \epsilon_y) \ \exp \ (i\nu\tau) \right\} \tag{2}$$

the gas lubrication equation can again be solved to obtain the perturbed forces, which can be written in the dimensionless form:

$$\frac{1}{p_a LD} \ (F_x, \ F_y) \ = R_e \left\{ (f_x, \ f_y) \ \exp \ (i\nu\tau) \right\} \tag{50}$$

The numerical values of $(f_x, \ f_y)$ depend on the particular geometry as well as the equilibrium operating conditions and the frequency of motion. For the present, we shall merely assume that the dynamic perturbation forces can be found for the entire range of ν and can be expressed as

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix} \tag{51}$$

The elements of the impedance matrix are complex quantities:

$$Z_{xx} = U_{xx} + iV_{xx} \quad \text{etc,}$$

<u>Case 1</u> $\epsilon_o \neq 0$, $Z_{xy} \neq 0$, $Z_{yx} \neq 0$.

The characteristic frequencies are the roots of

$$
\begin{aligned}
0 &= W_1(\nu_o) \\
&= \frac{\left[(U_{xx}-U_{yy})V_{xx} + U_{xy}V_{yx} + U_{yx}V_{xy}\right]\left[-(U_{xx}-U_{yy})V_{yy} + U_{xy}V_{yx} + U_{yx}V_{xy}\right]}{(V_{xx}+V_{yy})^2} \\
&\quad - V_{xx}V_{yy} - U_{xy}U_{yx} + V_{xy}V_{yx}
\end{aligned}
\tag{17}
$$

The critical mass is

$$
m_o = \frac{U_{xx}V_{yy} + U_{yy}V_{xx} - U_{xy}V_{yx} - U_{yx}V_{xy}}{\nu_o^2 (V_{xx}+V_{yy})}
\tag{16}
$$

An infinitessimal mass increment above m_o would constitute an unstable system if

$$
\left[2m_o\nu_o^2 - (U_{xx}+U_{yy})\right]\left[m_o\nu_o^2 \frac{\partial}{\partial\nu}(V_{xx}+V_{yy}) - \frac{\partial}{\partial\nu}(U_{xx}V_{yy}+V_{xx}U_{yy}-U_{xy}V_{yx}-V_{xy}U_{yx})\right]
$$

$$
+ (V_{xx}+V_{yy})\left[m_o\nu_o^2 \frac{\partial}{\partial\nu}(U_{xx}+U_{yy}) - \frac{\partial}{\partial\nu}(U_{xx}U_{yy}-V_{xx}V_{yy}-U_{xy}U_{yx}+V_{xy}V_{yx})\right]
$$

$$> 0$$

at $\nu = \nu_o$; and vice versa.

<u>Case 2</u> $Z_{xy} = Z_{yx} = 0$

The two degrees of freedom (x, y) are uncoupled.

The characteristic frequencies are the roots of

x - mode $\qquad\qquad$ $V_{xx}(\nu_o) = 0$

$$\text{(28)}$$

y - mode $\qquad\qquad$ $V_{yy}(\nu_o) = 0$

The critical mass is

x - mode $\qquad\qquad$ $m_o = \dfrac{1}{\nu_o^2} \left\{ \begin{array}{c} U_{xx} \\ \\ U_{yy} \end{array} \right\}$ $\qquad$ (29)

y - mode

An infinitessimal mass increment above m_o would constitute an unstable system if

x - mode $\qquad\qquad$ $\dfrac{\partial}{\partial \nu} \left\{ \begin{array}{c} V_{xx} \\ \\ V_{yy} \end{array} \right\} > 0$

y - mode

at $\nu = \nu_o$; and vice versa.

<u>Case 3</u> $Z_{xx} = Z_{yy} = Z_{\parallel}$, $Z_{xy} = - Z_{yx} = Z_{\perp}$

Generally, this corresponds to an unloaded journal bearing possessing rotational symmetry. $Z_{\parallel}$ and $Z_{\perp}$ are directly related to the small eccentricity, steady-whirl, radial and tangential forces, of the journal bearing with whirl ratio $\alpha' = \pm \nu$ (see Fig. 2).

The characteristic whirl ratio is determined by

$$\frac{1}{\epsilon} F_T(\alpha'_o) = 0 \tag{48}$$

The critical mass is

$$m_o = \frac{F_R(\alpha'_o)}{\epsilon {\alpha'_o}^2} \tag{49}$$

An infinitessimal mass increment above m_o would constitute an unstable system if

$$\frac{1}{\epsilon} \frac{\partial F_T}{\partial \alpha'} < 0$$

at

$\alpha' = \alpha'_o$; and vice versa.

EXAMPLES

In Ref. 13, the stability of an infinitely long, self-acting, plain, cylindrical journal bearing was treated. The procedure used to find the characteristic frequency and the critical mass was essentially the same as Case 1 above. The method to determine which side of the neutral state is unstable was different. Since there is no contradiction in fundamental principle between the two methods, one would not expect the final results to be different.

Case 2 is a one-degree-of-freedom problem. The corresponding stability criteria was first derived in Ref. 19, which also gives an example.

Two examples for Case 3 will be considered below.

Example 1. Unloaded, Plain, Cylindrical, Self-Acting, Journal Bearing

The steady-whirl results of this bearing was given in Ref. 25. They are reproduced in Figs. 3 and 4. The tangential bearing force vanishes at $\Lambda^* = 0$ or $\alpha'_o = 0.5$ for all Λ and L/D. The radial bearing force is also always zero there. Therefore, the critical mass is zero. Since, at $\Lambda^* = 0$,

$$\frac{1}{\epsilon}\frac{\partial F_T}{\partial \Lambda^*} > 0$$

$$\therefore \quad \frac{1}{\epsilon}\frac{\partial F_T}{\partial \alpha'} = - \Lambda \; \frac{1}{\epsilon}\frac{\partial F_T}{\partial \Lambda^*} < 0.$$

Since all finite rotor masses are larger than the zero critical mass, this bearing is always unstable. This is the infamous half-frequency whirl.

Example 2. Unloaded, Herringbone-Grooved, Cylindrical, Self-Acting Journal Bearing

A schematic of this bearing is shown in Fig. 5, its steady-whirl analysis was given in Ref. 21. Results of one such bearing for a range of Λ are shown in Figs. 6 and 7. The value of α_o', at which the tangential force, $\frac{1}{\epsilon}F_T$, vanishes, is about 0.5 when Λ is small; it increases with Λ and is about 1.7 at $\Lambda = 50$. At α_o', the radial force, $\frac{1}{\epsilon}F_R$, has some positive value, which also increases with Λ. The slope of $\frac{1}{\epsilon}F_T$ at α_o' is always negative. Therefore, masses larger than

$$m_o(\Lambda) = \frac{f(\Lambda, \alpha_o')}{(\alpha_o')^2}$$

are unstable, while smaller masses are stable. This results in a stability map as shown in Fig. 8.

CONCLUSIONS

1. The response of a gas bearing due to the perturbation of a periodic motion can be used to determine the stability map of the bearing.

2. The usual concepts of stiffness and damping can be generalized to include frequency dependence for either single-degree-of-freedom or coupled systems.

3. The condition of neutral stability exists at the characteristic frequency where the effective system damping vanishes and when the mass (critical mass) is in resonance with the effective system stiffness.

4. Mass increment above the critical mass is unstable if the effective system damping increases with frequency and conversely.

5. The stability map of unloaded journal bearings can be constructed from their steady-whirl results.

6. The spectral analysis provides a satisfactory means to separate the solution of the lubrication equation to that of the entire dynamical system.

ACKNOWLEDGEMENTS

NOMENCLATURE

C	Bearing Clearance
D	Bearing Diameter
e_o	Displacement of Journal Center From Bearing Center
e_x, e_y	Components of Rotor Displacement from its Equilibrium Position
f	Column Vector with the Components f_x and f_y
f, g	Real and Imaginary Parts of F
f_x, f_y	Normalized Complex Modulii of (F_x, F_y)
F	Normalized Complex Bearing Stiffness for Steady Whirl (Normalization Factor is $p_a LD$)
F_c	Conjugate of F
F_R	Normalized Radial Bearing Force, ϵf
F_T	Normalized Tangential Bearing Force, ϵg
F_x, F_y	Components of the Perturbed Bearing Force
I	Idemfactor or Identity Matrix
L	Bearing Length
m	$\dfrac{MC\omega^2}{p_a LD}$, Normalized Rotor Mass
M	Rotor Mass
p_a	Ambient Pressure
q, r	Functions
R	Bearing Radius
s	Normalized Complex Exponential Coefficient
t	Time
u	Real Part of z

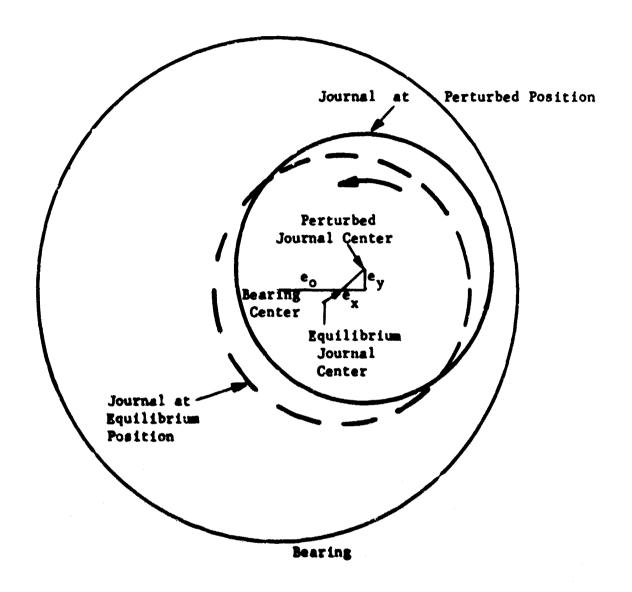|	
U	Real part of Z
U_{xx}, U_{xy}, etc.	Real Parts of Z_{xx}, Z_{xy}, etc.
v	Imaginary Part of z
V	Imaginary Part of Z
V_{xx}, V_{xy}, etc.	Imaginary Parts of Z_{xx}, Z_{xy}, etc.
W	Characteristic Determinant
W_1	Real Part of W
W_2	Imaginary Part of W
z	$-ms^2$, Normalized Effective System Impedance
Z	Normalized Impedance Matrix of the Bearing
Z_{xx}, Z_{xy}, etc.	Components of the Normalized Impedance Matrix
$z_{\parallel}$	Normalized Parallel or Colinear Impednace
$z_{\perp}$	Normalized Perpendicular or Cross Impednace
$(\ )_o$	Referring to the Condition of Neutral Stability or the Critical Condition
$\delta(\ )$	Infinitessimal Deviation of $(\ )$ from its Critical Value
α'	Whirl Ratio, Ratio of Angular Whirl Speed to ω
ϵ	Column Vector with the Components ϵ_x and ϵ_y; also $\sqrt{(\epsilon_x)^2+(\epsilon_y)^2}$
ϵ_x, ϵ_y	e_x/C, e_y/C
λ	Real Part of s
Λ	$\dfrac{6\mu\omega}{P_a}(\dfrac{R}{C})^2$, Compressibility Number
Λ^*	$\Lambda(1-2\alpha')$
μ	Viscosity
ν	Imaginary Part of s, Normalized Circular Frequency of Oscillation
ξ, η	Functions
τ	ωt, Normalized Time
ω	Rotor Rotational Speed

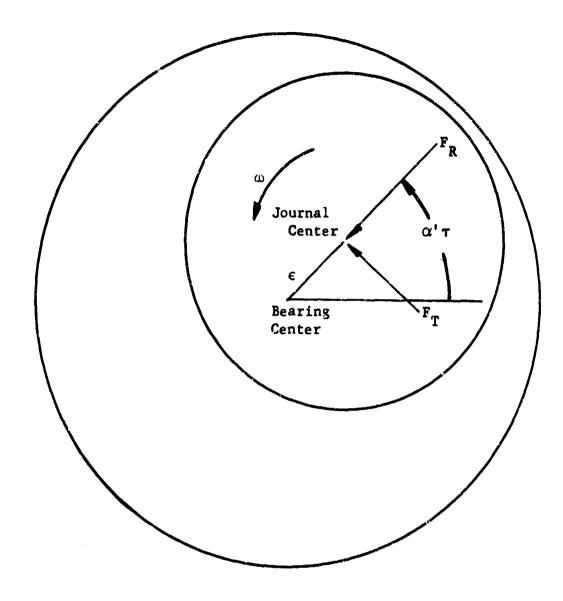# LIST OF FIGURES

Fig. 1   **Perturbation of Journal from Equilibrium
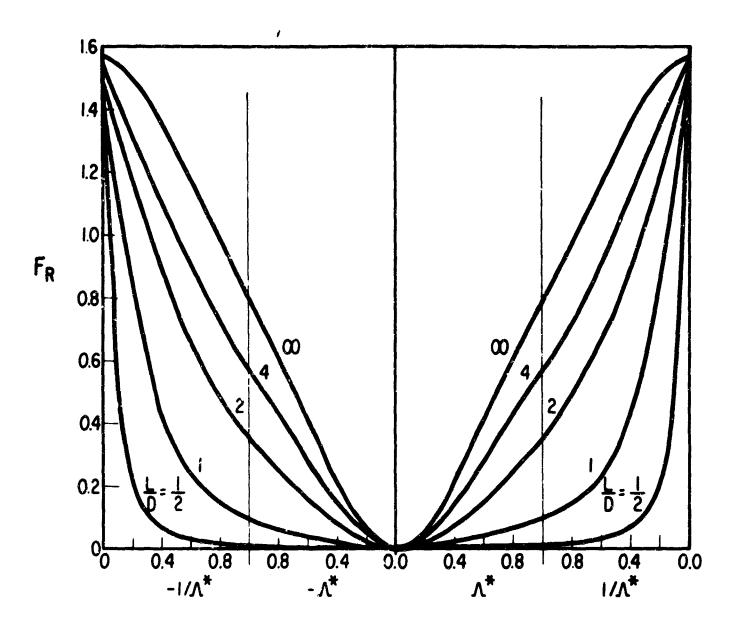Position**

Fig. 2   Journal Undergoing Steady Whirl

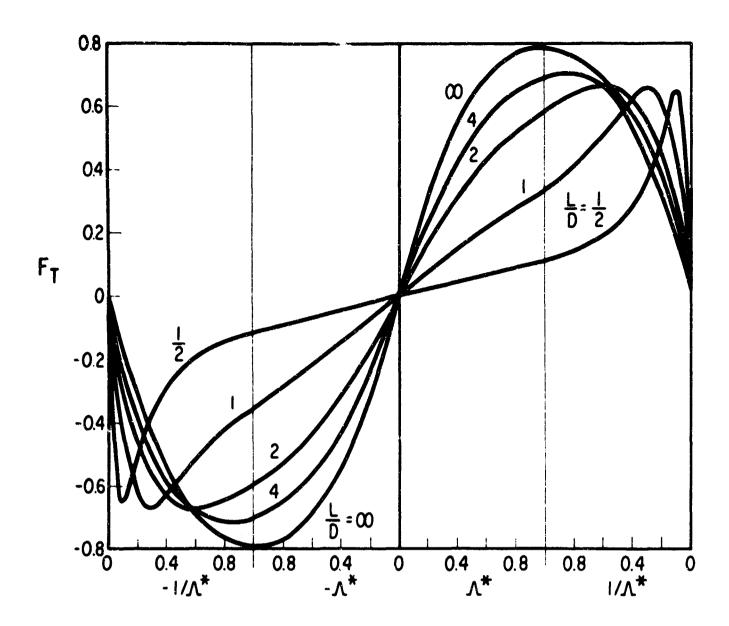Fig. 3   Small Perturbation Radial Force, Plain Journal Bearing

Fig. 4   Small Perturbation Tangential Force, Plain Journal Bearing
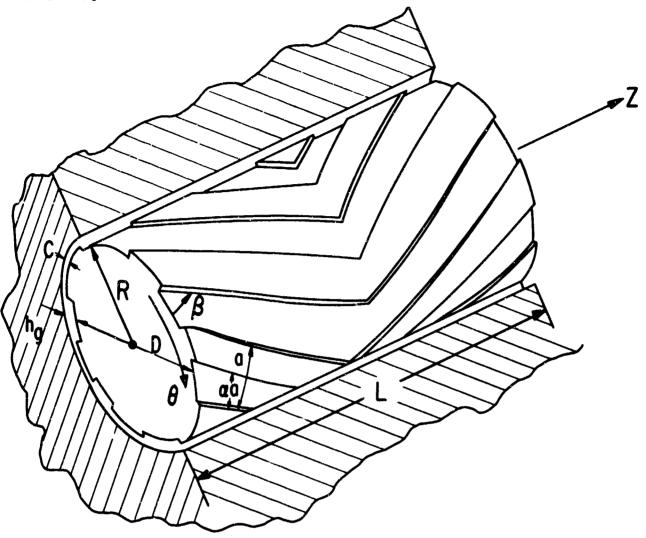
$\alpha = 0.5$
$\beta = 32.8°$
$hg/C = 2.1$
$L/D = 1.0$

Fig. 5   Herringbone-Grooved Journal Bearing
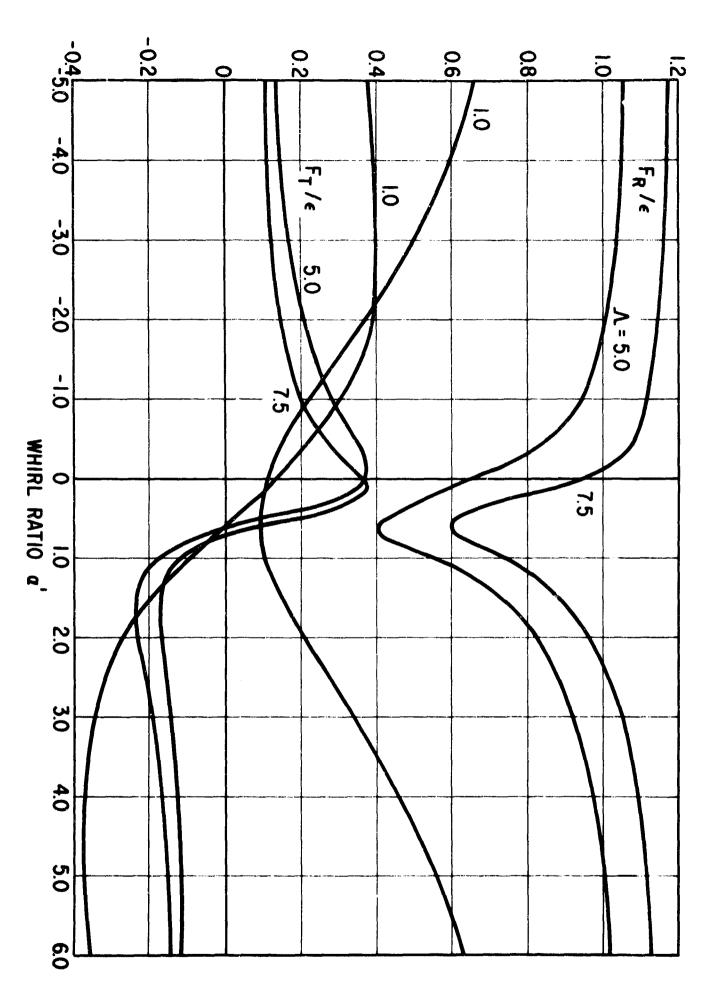
Fig. 6 Steady Whirl Forces of Herringbone-Grooved Journal Bearing — Small Λ

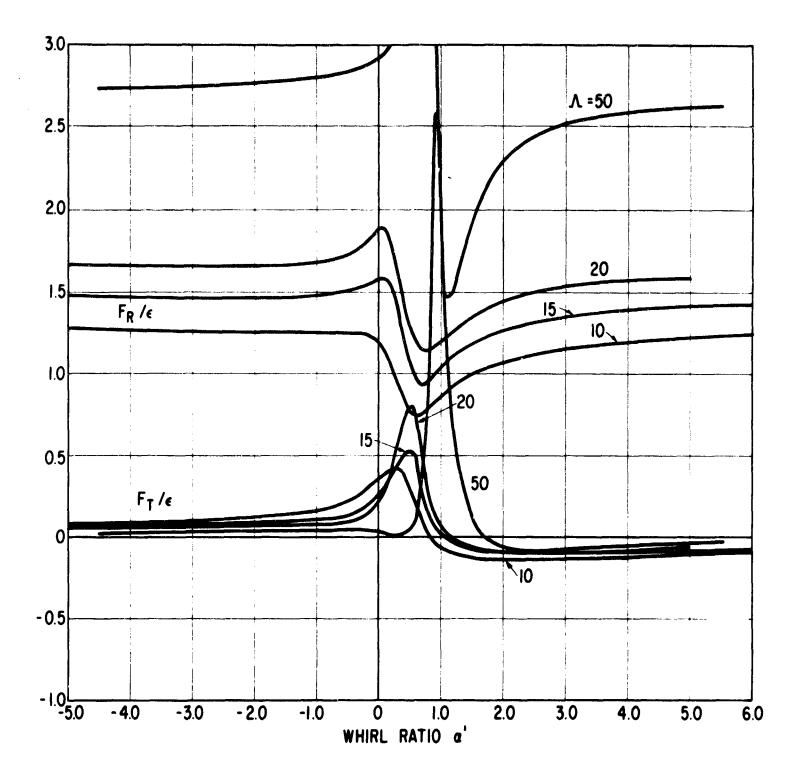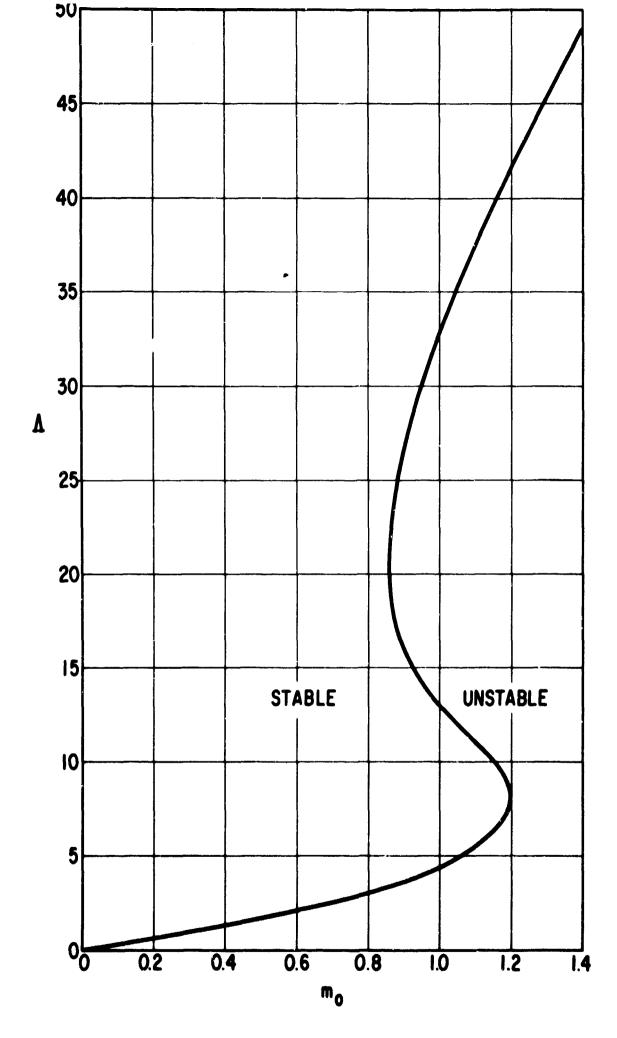Fig. 7 Steady Whirl Forces of Herringbone-Grooved Journal Bearing — Large Λ

Fig. 8   Stability Map of Herringbone-Grooved Journal Bearing

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